SOCI 102 – Exam 2 Study Guide Using Sample Exam

1.   Of the following z-score values, which one represents the location closest to the mean?
a.   z = +0.50
b.   z = +1.00
c.   z = −1.00
d.   z = −2.00

ANSWER: Since the mean of a distribution has a z-score value of 0, that means that the larger the absolute value (without regard for the + or -) of a z-score, the further it must be from the mean of 0. So z-scores with small absolute values are close to the mean, and z-scores with large absolute values are further from the mean. This question asks about which is closest, so the answer must be the one with the smallest absolute value, which is +0.05 or a.

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2.   For a population with $\mu = 80$ and $\sigma = 10$, what is the z-score corresponding to X = 70?
a.   −10
b.   −1.00
c.   +1.00
d.   +10

ANSWER: This is a straight conversion using the formula for calculating z-scores: $z = \dfrac{X - \mu}{\sigma}$

For this problem we would get $z = \dfrac{70 - 80}{10} = \dfrac{-10}{10} = -1$ which is choice b.

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3.   For a population with $\mu = 60$ and $\sigma = 8$, what is the X value corresponding to z = −0.50?
a.   −4
b.   56
c.   64
d.   59.5
 solve for is X instead of z.

The formula $z = \dfrac{X - \mu}{\sigma}$ needs to be solved for X, so we get $X = (\sigma * z) + \mu$

For this problem we get $X = (8 * -0.50) + 60 = 56$ which is choice b

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4.   Last week Sarah had exams in Math and in Spanish. On the Math exam, the mean was $\mu = 30$ with $\sigma = 5$, and Sarah had a score of X = 45. On the Spanish exam, the mean was $\mu = 60$ with $\sigma = 6$ and Sarah had a score of X = 65. For which class should Sarah expect the better grade?
a.   Math
b.   Spanish
c.   The grades should be the same because the two exam scores are in the same location.
d.   There is not enough information to determine which is the better grade.

ANSWER: Sarah is having a hard time because she is comparing apples with oranges. Imagine you have 1000 sasi and 500 yuyi (imaginary currencies), which is worth more? Hard to tell without knowing how much they represent of a common currency. If 1 sasi was worth $1.00 and 1 yuyi $.50, then you could easily compare and see that you have $1000 in sasi and only $250 in yuyi. Regardless of what you are trying to compare (money or test scores) you need a common measure, which is the z-score. So, if Sarah converts her scores into z-scores, she will be able to compare the two test scores directly.

Let's do the Math exam first. $z = \dfrac{X - \mu}{\sigma} = \dfrac{45 - 30}{5} = \dfrac{15}{5} = 3$ Sarah got a +3.00 z-score on the Math test

Now let's do the Spanish exam. $z = \dfrac{X - \mu}{\sigma} = \dfrac{65 - 60}{6} = \dfrac{5}{6} = 0.83$ Sarah got a $+0.83$ z-score on the Spanish test
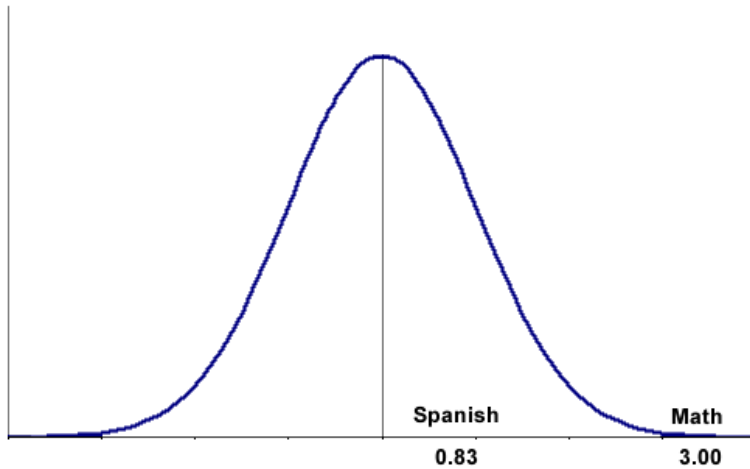
Sarah did much better on the Math test than on the Spanish test, so the answer is a.

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5.   An introductory psychology class has 9 freshman males, 15 freshman females, 8 sophomore males, and 12 sophomore females. What is the probability of randomly selecting a male from this group?

a. 9/24
b. 8/20
c. 17/20
d. 17/44

ANSWER: Basic principle is to identify how many possible outcomes (A) there are, and then to identify how many of those would match (B) some criterion. So the probability of tossing a coin and getting heads becomes (B) 2 possible outcomes, (B) only 1 outcome is heads. The formula is to divide the number of matching outcomes by the number of possible outcomes. $p = \dfrac{hits}{total}$

This question is a little more complicated, but the same basic principle is what we will be using to solve it. Lets think of the data this way:

	freshman	sophomore	Row Total
Male	9	8	17
female	15	12	27
Col. Total	24	20	Total = 44

For this question there are 44 ways to select someone, and 17 of them would be male, so $p = \dfrac{17}{44}$, choice d.

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6.   What proportion of a normal distribution is located in the tail beyond a z-score of $z = -1.50$?
a. −0.0668
b. −0.9332
c. 0.0668
d. 0.9332

ANSWER: Let's begin by drawing a picture to clarify what the question is asking. We are looking for the gray area in the picture below:
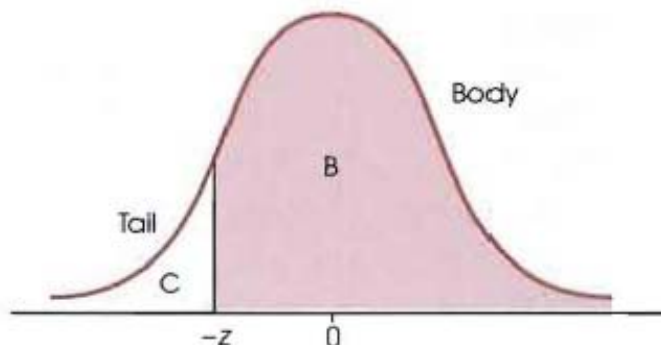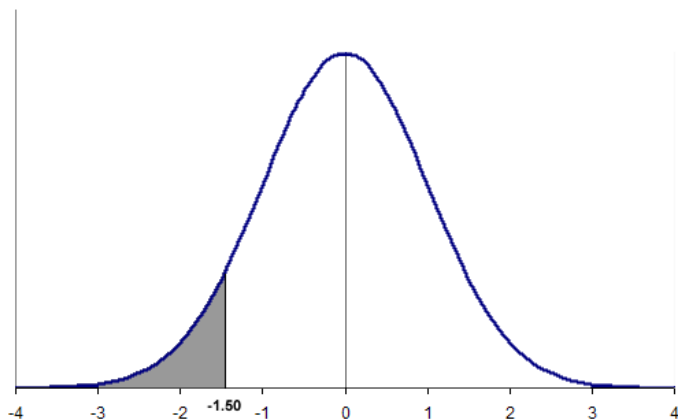
This area corresponds to the C column in the table on page 527. If we look for the row where z = 1.50, we will get our answer.

(A) z	(B) Proportion in Body	(C) Proportion in Tail	(D) Proportion Between Mean and z	(A)
1.50	.9332	.0668	.4332	2.0
1.51	.9345	.0655	.4345	2.0
1.52	.9357	.0643	.4357	2.0
1.53	.9370	.0630	.4370	2.0

So the value for the C column is .0668 which is choice c.

Note: two of the choices (a and b) are negative. Proportions can't be negative.

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7.   For a normal distribution, what z-score value separates the lowest 20% of the distribution from the highest 80%?
a.   z = 0.20
b.   z = 0.80
c.   z = 0.84
d.   z = −0.84

ANSWER: Let's begin by drawing a picture to clarify the question. We are looking for the z score in the picture below:
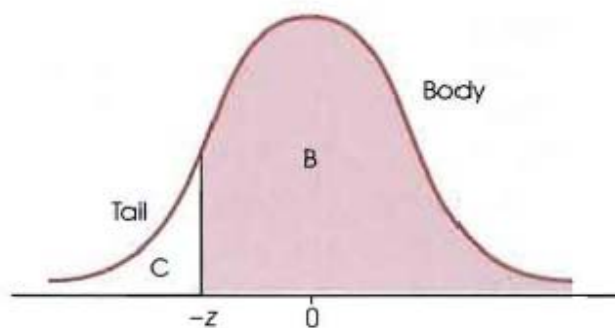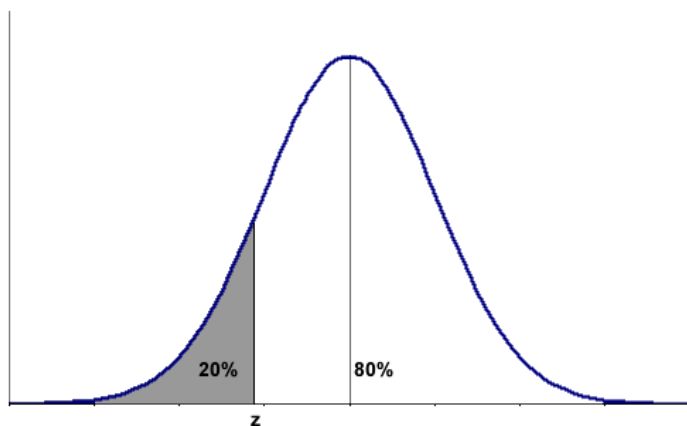
If we can find a C column entry close to .2000 (or a B column entry close to .8000) we have our answer.

| 0.82 | .7939 | .2061 | .2939 | |
0.83	.7967	.2033	.2967	
0.84	.7995	.2005	.2995	
0.85	.8023	.1977	.3023	
0.86	.8051	.1949	.3051	

The closest value to .2000 in column C is .2005 for a z value of 0.84. Since we are looking for the value of a z-score below the mean, we must make our answer a negative value, so we get – 0.84, or choice d.

Note: Choice d was the only negative answer.

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8.   A normal distribution has a mean of $\mu = 24$ with $\sigma = 3$. What score is needed to place in the top 14% of the distribution?
a.   20.76
b.   27.24
c.   25.08
d.   24.42

ANSWER: Let's begin by drawing a picture to clarify the question. We are looking for the z score in the picture below:
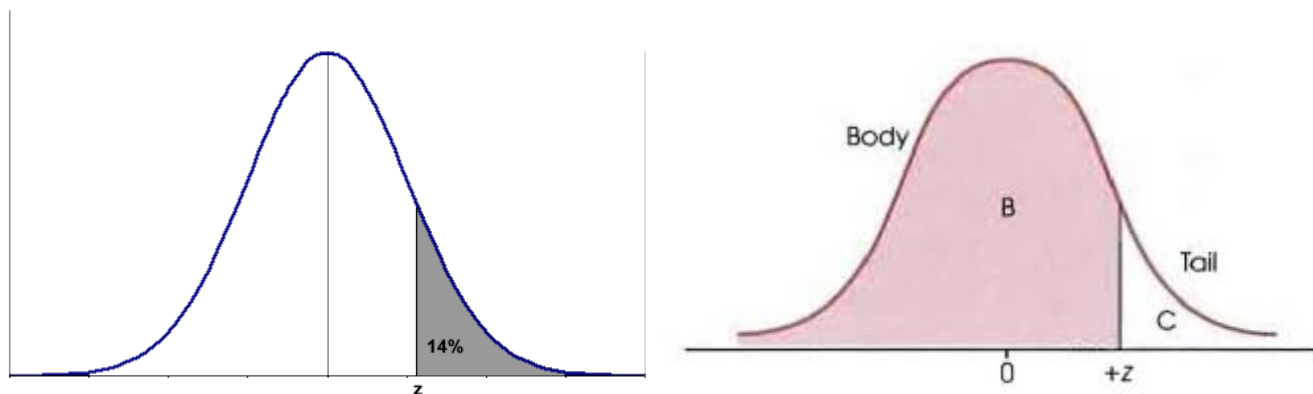
If we can find a C column entry close to .1400 we have our answer.

1.08	.8599	.1401	.3599
1.09	.8621	.1379	.3621

The closest value to .1400 in column C is .1401 for a z value of 1.08. Since we are looking for the value of a z-score above the mean, we must make our answer a positive value, so we get +1.08. Now that we know the z-score.

The formula $z = \dfrac{X - \mu}{\sigma}$ needs to be solved for X, so we get $X = (\sigma * z) + \mu = (3*1.08) + 24 = 3.24 + 24 = 27.24$

which is choice b.

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9.   The standard deviation of the distribution of sample means is called _____.
a.   the expected value of M
b.   the standard error of M
c.   the sample mean
d.   the central limit mean

ANSWER: b

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10.   Which of the following would produce a standard error of 3 points?
a.   n = 4 scores from a population with $\sigma = 6$
b.   n = 9 scores from a population with $\sigma = 9$
c.   n = 16 scores from a population with $\sigma = 12$
d.   All three samples would produce a standard error of 3 points.

ANSWER: The formula for a Standard Error is $\sigma_M = \dfrac{\sigma}{\sqrt{n}}$, so the choices would produce standard errors of:

Choice a: $\sigma_M = \dfrac{\sigma}{\sqrt{n}} = \dfrac{6}{\sqrt{4}} = 3$

Choice b: $\sigma_M = \dfrac{\sigma}{\sqrt{n}} = \dfrac{9}{\sqrt{9}} = 3$

Choice c: $\sigma_M = \dfrac{\sigma}{\sqrt{n}} = \dfrac{12}{\sqrt{16}} = 3$

All combinations generate a value of 3, so the answer is d.

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11.   A sample of n = 25 scores is determined to have a standard error of 2 points. What is the standard deviation for the population from which the sample was obtained?
a. 2
b. 2/5
c. 10
d. 50

ANSWER: If $\sigma_M = \dfrac{\sigma}{\sqrt{n}}$, then $\sigma_M * \sqrt{n} = \sigma$ or to make it easier to use, $\sigma = \sigma_M * \sqrt{n}$

So we enter our data and $\sigma = 2 * \sqrt{25} = 2 * 5 = 10$, which is choice c.

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12.   If sample size (n) is held constant, the standard error will _____ as the population variance increases.
a. increase
b. decrease
c. stay constant
d. cannot answer with the information given

ANSWER: Let's see what would happen by plugging some values into the Standard Error formula.

Lets use n = 25 and see what value we get for σ_M from a small value of σ, and from a large value of σ.

Small variance (small value for σ): $\sigma_M = \dfrac{5}{\sqrt{25}} = \dfrac{5}{5} = 1$

Large variance (large value for σ): $\sigma_M = \dfrac{500}{\sqrt{25}} = \dfrac{500}{5} = 100$

It appears that as variance increases, the standard error increases also, so the correct answer is a.

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13.   The critical region for a hypothesis test consists of _____.
a. outcomes that have a very low probability if the null hypothesis is true
b. outcomes that have a high probability if the null hypothesis is true
c. outcomes that have a very low probability whether or not the null hypothesis is true
d. outcomes that have a high probability whether or not the null hypothesis is true

ANSWER: The critical region is the same thing as the rejection region. If the null hypothesis is true, we would expect sample means to fall close to the population mean, so those that fell outside of that area (in the rejection region) would be the unlikely sample means.
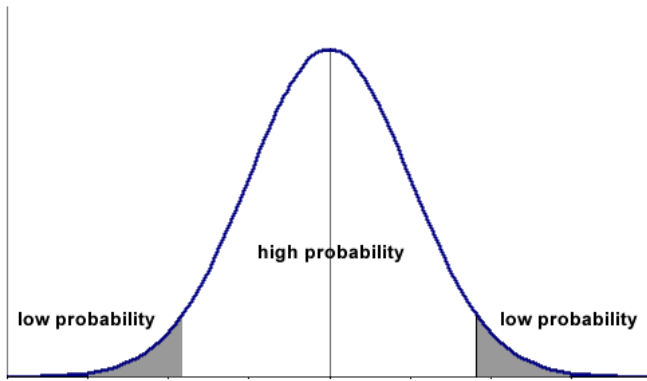
Correct choice is a.

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14.   In general, increasing the alpha level (for example from .01 to .05) will _____.
a.   increase the likelihood of rejecting the null hypothesis
b.   decrease the likelihood of rejecting the null hypothesis
c.   increase the standard error
d.   decrease the standard error

ANSWER: alpha is the probability of rejecting the null hypothesis when we should not (Type I error). Increasing its value will therefore increase that probability. Correct choice a.

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15.   In a hypothesis test, an extreme z-score value, like $z = +3$ or $z = +4$, _____.
a.   is probably in the critical region
b.   means that you should probably reject the null hypothesis
c.   is strong evidence of a statistically significant effect
d.   All of the other options are correct.

ANSWER: Correct answer is d.

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16.   A Type I error means that a researcher has _____.
a.   falsely concluded that a treatment has an effect
b.   correctly concluded that a treatment has no effect
c.   falsely concluded that a treatment has no effect
d.   correctly concluded that a treatment has an effect

ANSWER: A Type I error is rejecting the null hypothesis when there was no effect. That is to conclude something happened, when it did not, a false positive. Correct answer is a.

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17.   If a treatment has a very small effect, then a hypothesis test evaluating the treatment effect is likely to _____.
a.   result in a Type I error
b.   result in a Type II error
c.   correctly reject the null hypothesis
d.   correctly fail to reject the null hypothesis

ANSWER: The bigger the effect, the easier it is to see. That means that for a small effect, you might conclude there is no effect (fail to reject H_0), when there really was an effect (reject H_0). This would be a false negative, also known as a Type II error, or choice b.

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18.   A population is known to have a mean of $\mu = 45$. If a researcher predicts that the experimental treatment will produce a *decrease* in the population mean, then the null hypothesis for a one-tailed test would state _____.

a. $\mu \geq 45$
b. $\mu \leq 45$
c. $M \leq 45$
d. $M \geq 45$

ANSWER: If we want to know if our treatment will result in a sample with a mean less than 45, our alternative hypothesis would look like H_1: $\mu < 45$. Since the null hypothesis is whatever the alternative hypothesis is not, we get H_0: $\mu \geq 45$.

Correct answer is a.

Note: Remember that the hypotheses are always stated in terms of the population, so they will use the Greek letters.

Short Answer

19.   For a population with μ = 70 and σ = 8, find the z-score that corresponds to each of the following X values.

ANSWERS:

X = 74

$$z = \frac{74 - 70}{8} = \frac{4}{8} = 0.50$$

X = 68

$$z = \frac{68 - 70}{8} = \frac{-2}{8} = -0.25$$

X = 78

$$z = \frac{78 - 70}{8} = \frac{8}{8} = 1.00$$

X = 70

$$z = \frac{70 - 70}{8} = \frac{0}{8} = 0.00$$

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20.   A normal distribution has a mean of μ = 28 with σ = 5. Find the scores associated with the following regions:
a.   the score needed to be in the top 41% of the distribution
b.   the score needed to be in the top 72% of the distribution
c.   the scores that mark off the middle 60% of the distribution

ANSWERS

(Answer for a) - the score needed to be in the top 41% of the distribution

41%
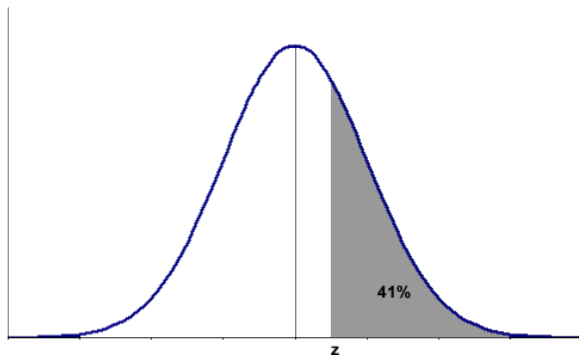
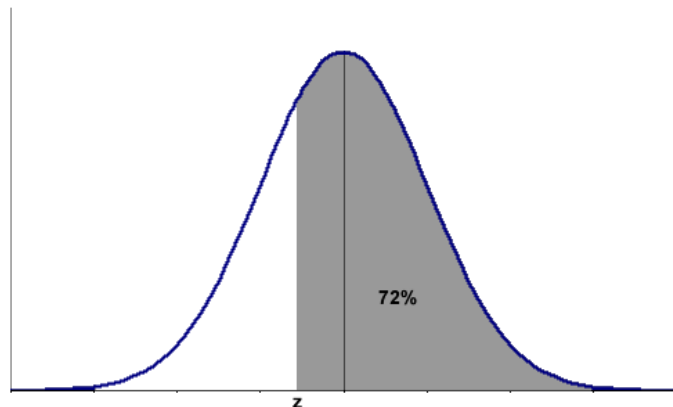Look for the closest entry to .4100 in the C column of the table on page 527.

0.20	.5793	.4207	.0793
0.21	.5832	.4168	.0832
0.22	.5871	.4129	.0871
0.23	.5910	.4090	.0910
0.24	.5948	.4052	.0948

Closest value is .4090, which corresponds to a z-score value of 0.23.

Converting this z-score back into the original scores gives us

$$X = (\sigma * z) + \mu = (5 * 0.23) + 28 = 1.15 + 28 = 29.15$$

(Answer for b) – the score needed to be in the top 72% of the distribution

72%

Look for the closest entry to .7200 in the B column of the table on page 527.

0.57	.7157	.2843	.2157
0.58	.7190	.2810	.2190
0.59	.7224	.2776	.2224
0.60	.7257	.2743	.2257
0.61	.7291	.2709	.2291

Closest value is .7190, which corresponds to a z-score value of 0.58. Because this z-score will be below the mean, we need to convert it into a negative z-score, or -0.58.

Converting this z-score back into the original scores gives us
$$X = (\sigma * z) + \mu = (5 * -0.58) + 28 = -2.90 + 28 = 25.10$$

(Answer for c) - the scores that mark off the middle 60% of the distribution

60%

D

0   z
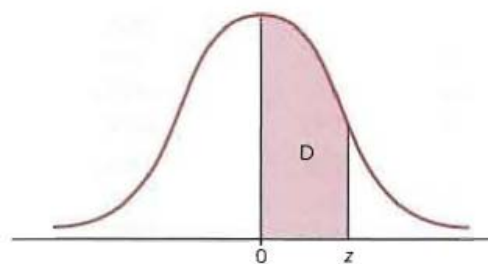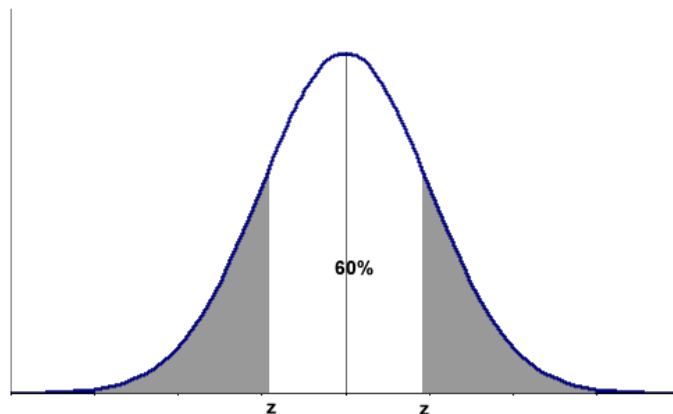
Since the curve is symmetrical, we can assume that of the middle 60%, half (30%) lies to the right of the mean, and the other half lies to the left of the mean. Also, because the curve is symmetrical, we can assume that once we find the value for z above the mean, the value for z below the mean will be the same, but negative.

For this problem, probably the best column to look under in the table on page 527 is column D. If we can find a value of D close to .3000, then that z value and its negative will be the scores that define that middle 60%.

0.83	.7967	.2033	.2967	1.33
0.84	.7995	.2005	.2995	1.34
0.85	.8023	.1977	.3023	1.35
0.86	.8051	.1949	.3051	1.36

The closest value to .3000 is .2995 which is associated with a z-score value of 0.84. Now we need to convert 0.84 and -0.84 into the original distribution.
+0.84 gives us $X = (\sigma * z) + \mu = (5 * 0.84) + 28 = 4.20 + 28 = 32.20$

-0.84 gives us $X = (\sigma * z) + \mu = (5 * -0.84) + 28 = -4.20 + 28 = 23.80$

So the scores that define the middle 60% are 23.80 and 32.20.

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21.   A normal distribution has $\mu = 40$ and $\sigma = 8$.
a.   Describe the distribution of sample means based on samples of n = 16 selected from this population.
b.   Of all the possible samples of n = 16, what proportion will have sample means greater than 42?
c.   Of all the possible samples of n = 16, what proportion will have sample means less than 39?

ANSWERS:

(Answer for a) - The distribution of sample means will have the same mean as the original distribution, and a standard error as its standard deviation. So it will have $\mu = 40$ and $\sigma_M = \dfrac{8}{\sqrt{16}} = \dfrac{8}{4} = 2$

(Answer for b) – We will need to calculate a z-score for a sample mean of 42, NOT A RAW SCORE OF 42. That means that instead of using the distribution of population scores, we need to use the sampling distribution. Let's sketch it out using the same mean as for the distribution of population scores, but using the standard error instead of the standard deviation:
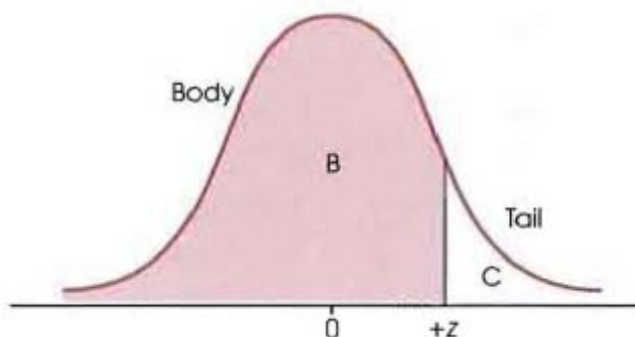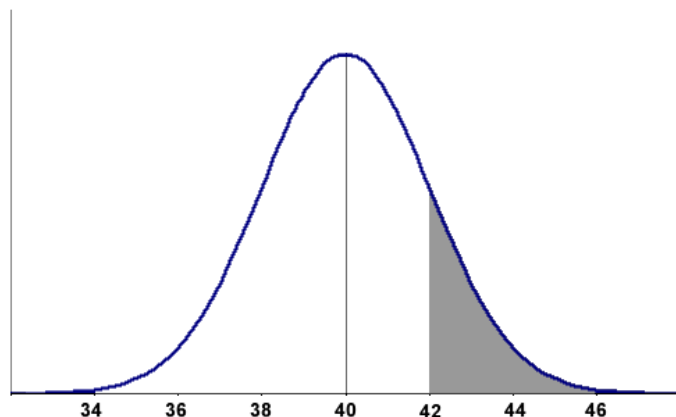
Looks like the C column from our table. So lets calculate a z-score for a score of 42, and look up the value for column C in our table to get our answer. Remember that we are using the sampling distribution, so we need to use the standard error instead of the standard deviation.

$$z = \frac{X - \mu}{\sigma_M} = \frac{42 - 40}{2} = \frac{2}{2} = 1.00$$

If we look up a z-score of 1.00 in our table, we get a C column value of 0.1587, so our answer is 0.1587.

(A) z	(B) Proportion in Body	(C) Proportion in Tail	(D) Proportion Between Mean and z
1.00	.8413	.1587	.3413
1.01	.8438	.1562	.3438

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22.   A study examines self-esteem and depression in children. A sample of 25 children with low self-esteem is given a standardized test for depression. The average score for the group is M = 93.3. For the general population, scores on the depression test form a normal distribution with a mean of $\mu = 90$ and a standard deviation of $\sigma = 15$. Do children with low self-esteem show significantly more depression? Use a one-tailed test with $\alpha = .05$.

ANSWERS: This problem is asking about the probability of selecting a sample of 25 children from a population of children with average scores of 90 and a standard deviation of 15 on this depression test, and getting a sample mean of 93.3. If it is likely to do so, we may not conclude that our low self-esteem sample is any different than non-low self-esteem children. If on the other hand the mean for our sample is unusually high, it would suggest that low self-esteem children do in fact have higher scores on this test than non-low self-esteem children because it would be unlikely to have selected such a sample unless they indeed had higher scores.

Step 1 – Set up your hypotheses

We want to know if the low self-esteem children have HIGHER scores. This is a one-tailed hypothesis.
$H_1 = \mu > 90$

After you set up the alternative hypothesis, the null hypothesis is anything not covered by the alternative hypothesis.

$H_0 = \mu \leq 90$

So we have

$H_0 = \mu \leq 90$
$H_1 = \mu > 90$

Step 2 – Calculate a Standard Error

Using the Standard Error formula, plug in the numbers for the population standard deviation, and our sample size.

$$\sigma_M = \frac{\sigma}{\sqrt{N}} = \frac{15}{\sqrt{25}} = \frac{15}{5} = 3.00$$

Step 3 – Calculate z_{obt} (the z we obtain from this test, often times we just drop the "obt")

Use the z_{obt} formula, and plug into it our sample mean, the population mean we are comparing it against, and the standard error we calculated in the last step.

$$z_{obt} = \frac{M - \mu}{\sigma_M} = \frac{93.3 - 90}{3} = \frac{3.3}{3} = 1.10$$

Step 4 – Determine z_{crit} (the critical value of z for rejecting H_0) and interpret the z_{obt} we just calculated.

In our class we will always use a level of significance of .05, and this is a one-tailed hypothesis with the upper tail critical because we are looking for our sample mean to be HIGHER than the population mean we are comparing it against. The critical value of z for a one-tailed test is 1.65, and since this is an upper tail critical test we need to make that a positive (+1.65).

Zcrit = **+1.65**

Zobt = **+1.11**
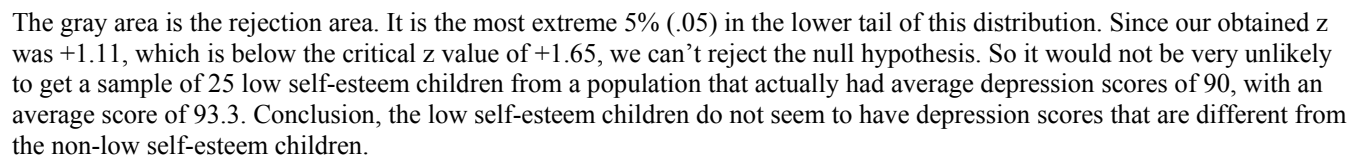
The gray area is the rejection area. It is the most extreme 5% (.05) in the lower tail of this distribution. Since our obtained z was +1.11, which is below the critical z value of +1.65, we can't reject the null hypothesis. So it would not be very unlikely to get a sample of 25 low self-esteem children from a population that actually had average depression scores of 90, with an average score of 93.3. Conclusion, the low self-esteem children do not seem to have depression scores that are different from the non-low self-esteem children.