z-score and Unit Normal Table Exercises

Exercise 1 - Mr. Jones has given an exam to his math class. Below you will find the scores for 6 of his students who took the exam. The way Mr. Jones gives grades is based on percentiles. Anybody's scoring above the 85th percentile gets an A, between the 70th and the 85th percentile students get a B, between the 55th and the 70th percentile students get a C, between the 40th and the 55th percentile students get a D, and below the 40th percentile students get an F.

Using their exam scores as a starting point, calculate a grade for each of the six students by first calculating a z-score, then using the z-score to calculate a percentile, and finally matching their percentile to a letter grade. The mean and standard deviation for the exam are given below the scores.

Student	Score	z-score	Percentile	Grade
Aida	54	_____	_____	_____
John	52	_____	_____	_____
Lenore	67	_____	_____	_____
David	78	_____	_____	_____
Justin	69	_____	_____	_____
Sarah	59	_____	_____	_____
Exam Mean	55			
Exam SD	7			

Exercise 2 - Miss Rodriguez was talking about grading to Mr. Jones, and liked the idea of giving letter grades based on percentiles. Since she has already given out four exams, she does not want to go back and recalculate each student's score individually. Instead, she will calculate the cut-off points for each of the four exams, give these values to her TA, and have him go back and change all the grades based on these values.

Below you are given the mean and standard aviation for each of Ms. Rodriguez's four exams. For each exam, calculate the minimum score needed to get a particular letter grade. Use the same criteria as Mr. Jones in the first exercise, above the 85th percentile is an A, between the 70th and the 85th percentiles is a B, between the 55th and the 70th percentiles is a C, between the 40th and the 55th percentiles is a D, and below the 40th percentile is an F.

			Minimum points for letter grade			
Exam	Mean	SD	A	B	C	D
Exam 1	55	5	_____	_____	_____	_____
Exam 2	58	7	_____	_____	_____	_____
Exam 3	62	8	_____	_____	_____	_____
Exam 4	45	4	_____	_____	_____	_____

Exercise 3 - Mr. Smith has 100 students in his class. The last exam he gave was normally distributed with a mean of 82 and a standard deviation of 10. Use this information to answer the following questions.

1. If Sue scored a 90, and John scored a 95, what is the probability of a student scoring more than Sue but less than John?

2. How many students scored above an 80?

3. What is the probability of a student scoring more than 70, but less than 90?

4. If the bottom 25% get an F, what would be the minimum score needed to get at least at D?

5. If only the top 10% get an A, what score would you need to get an A on the exam?

6. If the middle 30% get a C, what would be the minimum and maximum scores a student could get to earn a C in the exam?

7. How many students scored between a 70 and an 85 on the exam?

8. What is the probability that a student scored less than 75 points on the exam?

9. What percentage of students scored between 70 and 80 points on the exam?

10. What proportion of students scored between 85 and 90 points on the exam?

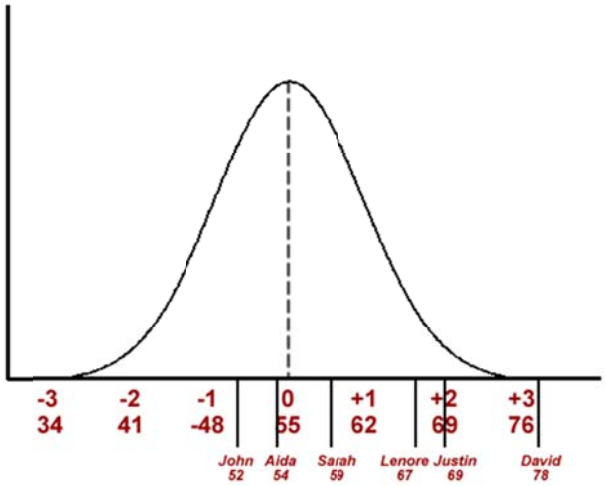
Exercise 4 - You are in charge of enrollment at a prestigious university. The university only accepts the top 10% of GRE scores, and uses these scores to organize students into 4 tiers to determine the order in which they can pick their classes. The top 5% get first choice, then the next 5%, and so on. You already know that the GRE has a mean of 500, and a standard deviation of 100. What GRE scores would define these four tiers?

Tier		Minimum Score
First	Top 2.5%	_____
Second	2.5%-5.0%	_____
Third	5.0%-7.5%	_____
Fourth	7.5%-10%	_____

Answers

Exercise 1 - To answer these questions you will need to convert each score into a z-score, and then use that value to look in the unit normal table for the percentage of the curve than lies below it. Your answers should be as follows:

Student	Score	z-score	Percentile
Aida	54	-0.14	44.43
John	52	-0.43	33.36
Lenore	67	1.71	95.64
David	78	3.29	99.95
Justin	69	2.00	97.72
Sarah	59	0.57	71.57
Exam Mean	55		
Exam SD	7		



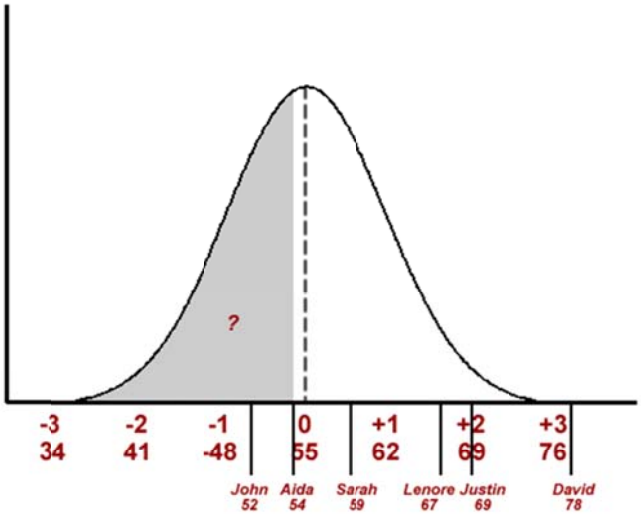
To answer these you should begin by drawing a distribution, and mapping each of the six students onto it like the image on the right.

Once you have the image made, the problem can be conceptualized a little easier. For example, looking at the image of the distribution on the left we would expect the z-score for Aida to be a little bit less than zero, and the percentile a little less than 50. Since each half of the distribution accounts for half of it, then a little less than half should be a little bit less than a percentile score of 50.

Now that you have drawn out the problem and you have a good idea of what values you should be expecting, you can start to calculate the precise values.

$$z = \frac{X - \mu}{\sigma} = \frac{54 - 55}{7} = \frac{-1}{7} = -0.14$$

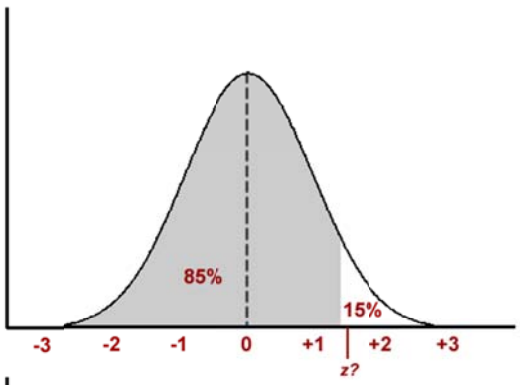
Once you have a z-score, you can look in the table to find the corresponding value for the area you are interested in. You should look for the value in the C column of the table for a corresponding z-score value of 0.14. Remember that the table does not use negative values. The value you get is 0.4443, which translates to a percentile of 44.43.



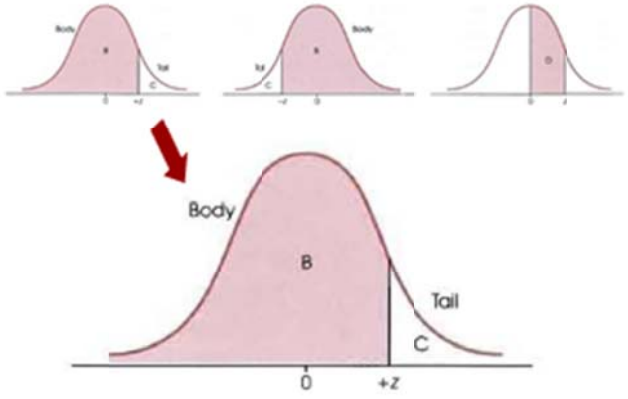
(A) z	(B) Proportion in Body	(C) Proportion in Tail	(D) Proportion Between Mean and z
0.10	.5398	.4602	.0398
0.11	.5438	.4562	.0438
0.12	.5478	.4522	.0478
0.13	.5517	.4483	.0517
0.14	.5557	.4443	.0557

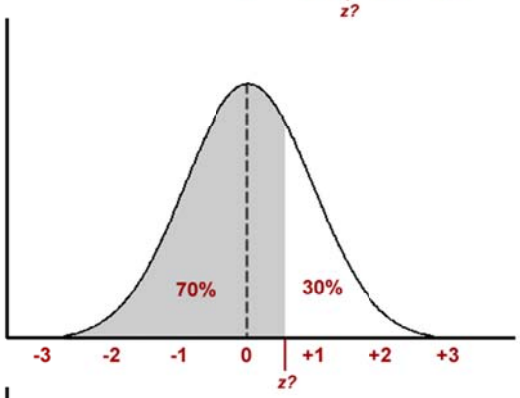
Exercise 2 – The easiest way to begin to answer these problems is to calculate the z-scores that define the percentiles of interest. In other words, we want to know what z-scores define the top 15%, the top 30%, the top 45%, and the top 60%. So let's sketch out a distribution and identify the areas we are interested in. Then we can calculate what z-scores would define those areas.

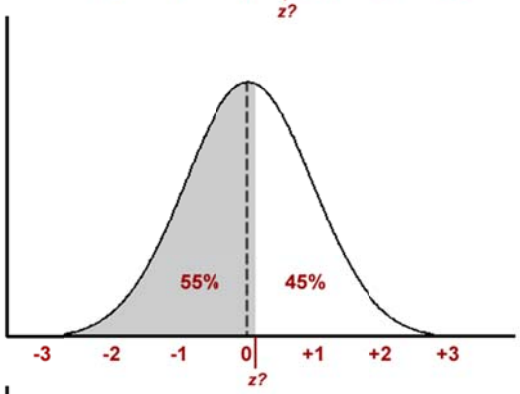
Grade	Percentile	z-score
A	85	1.04
B	70	0.52
C	55	0.13
D	40	-0.25



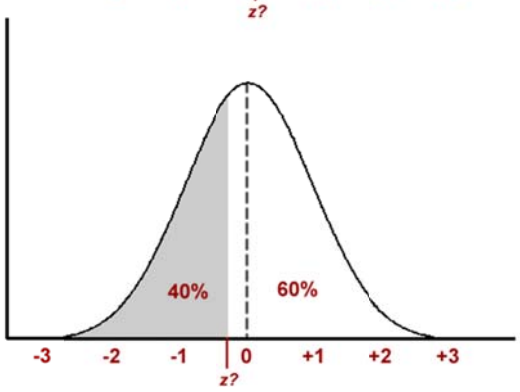
To calculate the first z-score we need to look in the units normal table for the value for z. Begin by comparing your sketch of the problem (top right image) to the images at the start of the table.







It should be easy to see that the problem we are trying to solve looks like the image on the top left of the top of the unit normal table. We want to find the value of z when C equals 15% or .1500.



(A) z	(B) Proportion in Body	(C) Proportion in Tail	(D) Proportion Between Mean and z
1.02	.8461	.1539	.3461
1.03	.8485	.1515	.3485
1.04	.8508	.1492	.3508
1.05	.8531	.1469	.3531

Once the z-scores are identified, the table can be completed by calculating the corresponding scores for each of the four exams.

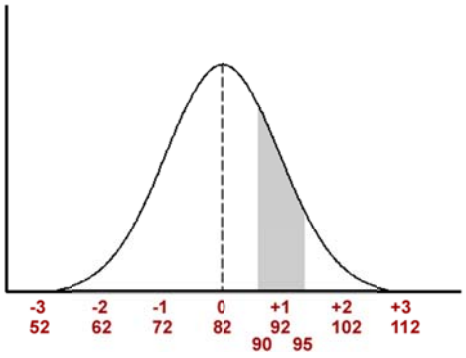
For Exam 1, the answers would be calculated as follows:

X = (z*σ) + μ = (1.04 * 5) + 55 = 5.20 + 55 = 60.20
X = (z*σ) + μ = (0.52 * 5) + 55 = 2.60 + 55 = 57.60
X = (z*σ) + μ = (0.13 * 5) + 55 = 0.65 + 55 = 55.65
X = (z*σ) + μ = (-0.25 * 5) + 55 = -1.25 + 55 = 53.75

			Minimum points for letter grade			
Exam	Mean	SD	A	B	C	D
Exam 1	55	5	60.20	57.60	55.65	53.75
Exam 2	58	7	65.28	61.64	58.91	56.25
Exam 3	62	8	70.32	66.16	63.04	60.00
Exam 4	45	4	49.16	47.08	45.52	44.00

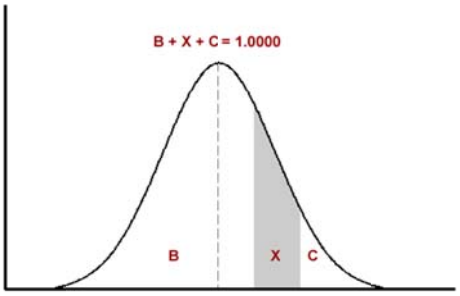
Exercise 3 – Begin by sketching out the question. Remember the mean was 82, and the standard deviation was 10.

1. If Sue scored and 90, and John scored a 95, what is the probability of a student scoring more than Sue but less than John?



The picture on the right illustrates the question. We're interested in the area shaded in gray, which is the area between a score of 90 and a score of 95.

First we need to calculate z-scores for scores of 90 and 95. Since the unit normal table only has z-scores, we need to have z-scores to be able to use it.



$$z = \frac{X - \mu}{\sigma} = \frac{90 - 82}{10} = \frac{8}{10} = 0.80$$

$$z = \frac{X - \mu}{\sigma} = \frac{95 - 82}{10} = \frac{13}{10} = 1.30$$

Once you know the value for the z-scores that correspond to 90 and 95, you can start to look for the area. This particular problem is a little bit tricky in that the area we want does not have the mean or the tail as one of its boundaries. There are actually several ways that we can solve this problem using our tools. Basically, we know that the whole distribution accounts for 100% of itself or 1.0000. If we can figure out the white areas, then the gray area must be one minus the white area.

if… B + X + C = 1.0000 then… X = 1.0000 – B – C

We can calculate the white area in two pieces, one the piece to the left of the gray area, and the other the piece to the right.

(A) z	(B) Proportion in Body	(C) Proportion in Tail	(D) Proportion Between Mean and z	(A) z	(B) Proportion in Body	(C) Proportion in Tail	(D) Proportion Between Mean and z
0.78	.7823	.2177	.2823	1.28	.8997	.1003	.3997
0.79	.7852	.2148	.2852	1.29	.9015	.0985	.4015
0.80	.7881	.2119	.2881	1.30	.9032	.0968	.4032
0.81	.7910	.2090	.2910	1.31	.9049	.0951	.4049
0.82	.7939	.2061	.2939	1.32	.9066	.0934	.4066

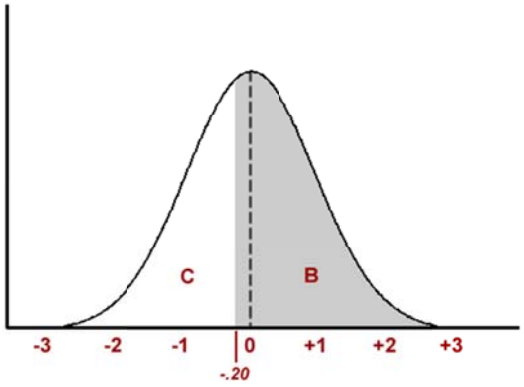
B = .7881; C = .0968 therefore X = 1.0000 - .7881 - .0968 = .1151

Answer - There is a .1151 probability of a student getting a score between 90 and 95.

2. How many students scored above an 80?

$$z = \frac{X - \mu}{\sigma} = \frac{80 - 82}{10} = \frac{-2}{10} = -0.20$$

(A) z	(B) Proportion in Body	(C) Proportion in Tail	(D) Proportion Between Mean and z
0.19	.5753	.4247	.0753
0.20	.5793	.4207	.0793
0.21	.5832	.4168	.0832



The proportion of the area above a z-score of -.20 is .5793. Since there are 100 students, there must be 100 * .5793 = 57.93 students who scored above 80 on the exam. (57.93% of 100 students is 57.93)

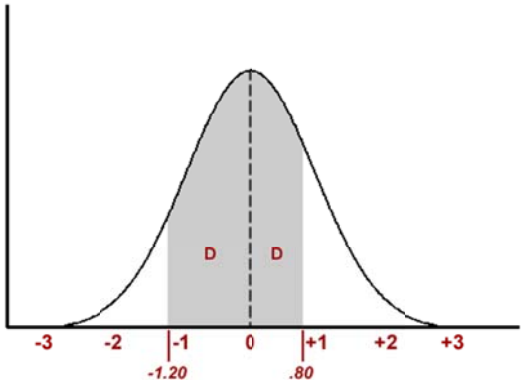
Answer – 58 students.

3. What is the probability of a student scoring more than 70, but less than 90?

$$z = \frac{X - \mu}{\sigma} = \frac{70 - 82}{10} = \frac{-12}{10} = -1.20$$

$$z = \frac{X - \mu}{\sigma} = \frac{90 - 82}{10} = \frac{8}{10} = 0.80$$

(A) z	(B) Proportion in Body	(C) Proportion in Tail	(D) Proportion Between Mean and z
1.19	.8830	.1170	.3830
1.20	.8849	.1151	.3849
1.21	.8869	.1131	.3869
0.79	.7852	.2148	.2852
0.80	.7881	.2119	.2881
0.81	.7910	.2090	.2910



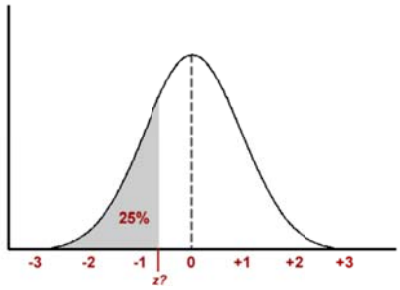
The combined area is equal to .3849 + .2881 = .6730

Answer – The probability that a student will score between 70 and 90 is .6730

4. If the bottom 25% get an F, what would be the minimum score needed to get at least a D?

(A) z	(B) Proportion in Body	(C) Proportion in Tail	(D) Proportion Between Mean and z
0.65	.7422	.2578	.2422
0.66	.7454	.2546	.2454
0.67	.7486	.2514	.2486
0.68	.7517	.2483	.2517
0.69	.7549	.2451	.2549

X = (z*σ) + μ = (0.67 * 10) + 82 = 6.70 + 82 = 88.70

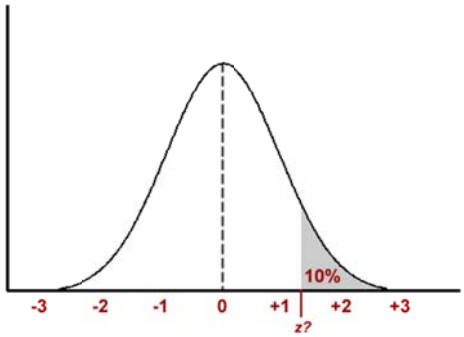


Answer – Students would need to get a score of at least 88.70.

5. If only the top 10% get an A, what score would you need to get me on the exam?

(A) z	(B) Proportion in Body	(C) Proportion in Tail	(D) Proportion Between Mean and z
1.27	.8980	.1020	.3980
1.28	.8997	.1003	.3997
1.29	.9015	.0985	.4015

X = (z*σ) + μ = (1.28 * 10) + 82 = 12.80 + 82 = 94.80



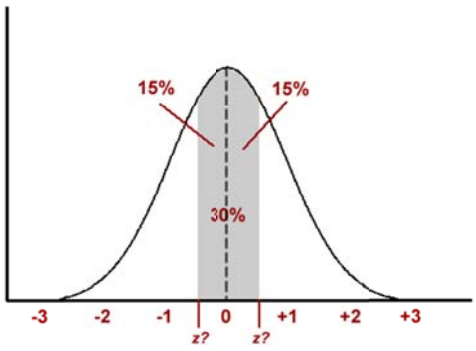
Answer - Students would need to get a score of at least 94.80.

6. If the middle 30% get a C, what would be the minimum and maximum scores a student could get to earn a C in the exam?

(A) z	(B) Proportion in Body	(C) Proportion in Tail	(D) Proportion Between Mean and z
0.38	.6480	.3520	.1480
0.39	.6517	.3483	.1517
0.40	.6554	.3446	.1554

X = (z*σ) + μ = (0.39 * 10) + 82 = 3.90 + 82 = 85.90

X = (z*σ) + μ = (-0.39 * 10) + 82 = -3.90 + 82 = 78.10

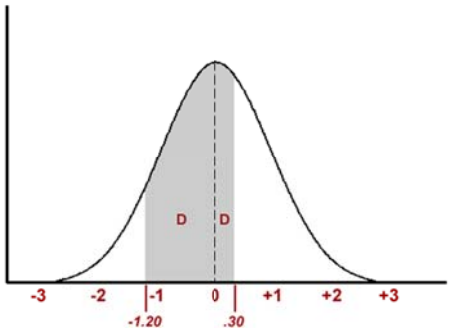


Answer – A student would need a score of at least 78.10 and not more than 85.90.

7. How many students scored between a 70 and an 85 on the exam?

$$z = \frac{X - \mu}{\sigma} = \frac{70 - 82}{10} = \frac{-12}{10} = -1.20$$

$$z = \frac{X - \mu}{\sigma} = \frac{85 - 82}{10} = \frac{3}{10} = 0.30$$



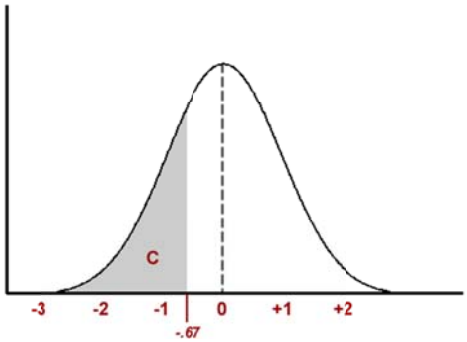
(A) z	(B) Proportion in Body	(C) Proportion in Tail	(D) Proportion Between Mean and z
0.29	.6141	.3859	.1141
0.30	.6179	.3821	.1179
0.31	.6217	.3783	.1217
1.19	.8830	.1170	.3830
1.20	.8849	.1151	.3849
1.21	.8869	.1131	.3869

Since the proportion of the distribution between a z-score of -1.20 and a z-score of .30 is (.1179 + .3849) .5029, and there are 100 students, 50.29% of 100 is 50.29 students.

Answer – 50 students.

8. What is the probability that a student scored less than 75 points on the exam?

$$z = \frac{X - \mu}{\sigma} = \frac{75 - 82}{10} = \frac{-7}{10} = -0.70$$



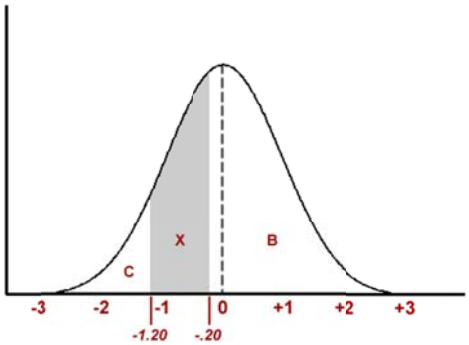
(A) z	(B) Proportion in Body	(C) Proportion in Tail	(D) Proportion Between Mean and z
0.66	.7454	.2546	.2454
0.67	.7486	.2514	.2486
0.68	.7517	.2483	.2517

Answer - The probability that a student will score below 70 is .2514.

9. What percentage of students scored between 70 and 80 points on the exam?

$$z = \frac{X - \mu}{\sigma} = \frac{70 - 82}{10} = \frac{-12}{10} = -1.20$$

$$z = \frac{X - \mu}{\sigma} = \frac{80 - 82}{10} = \frac{-2}{10} = -0.20$$



(A) z	(B) Proportion in Body	(C) Proportion in Tail	(D) Proportion Between Mean and z
0.19	.5753	.4247	.0753
0.20	.5793	.4207	.0793
0.21	.5832	.4168	.0832
1.19	.8830	.1170	.3830
1.20	.8849	.1151	.3849
1.21	.8869	.1131	.3869

B = .5793; C = .1151 therefore X = 1.0000 - . 5793- . 1151 = .3056 or 30.56%

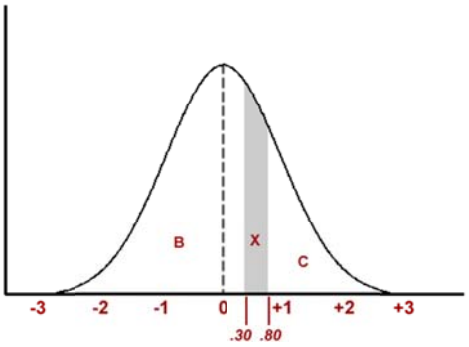
Answer - 30.56% of students scored between 70 and 80 points on the exam.

10. What proportion of students scored between 85 and 90 points on the exam?

$$z = \frac{X - \mu}{\sigma} = \frac{85 - 82}{10} = \frac{3}{10} = 0.30$$

$$z = \frac{X - \mu}{\sigma} = \frac{90 - 82}{10} = \frac{8}{10} = 0.80$$

(A) z	(B) Proportion in Body	(C) Proportion in Tail	(D) Proportion Between Mean and z
0.29	.6141	.3859	.1141
0.30	.6179	.3821	.1179
0.31	.6217	.3783	.1217
0.79	.7852	.2148	.2852
0.80	.7881	.2119	.2881
0.81	.7910	.2090	.2910

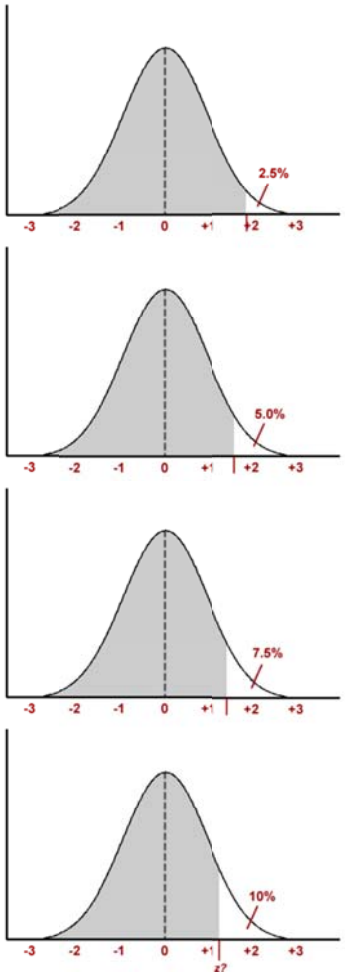


B = .6179; C = .2119 therefore X = 1.0000 - . 6179 - . 2119 = .1702

Answer – The proportion of students who scored between 85 and 90 points on the exam was.1702.

Exercise 4 -

(A) z	(B) Proportion in Body	(C) Proportion in Tail	(D) Proportion Between Mean and z
1.27	.8980	.1020	.3980
1.28	.8997	.1003	.3997
1.29	.9015	.0985	.4015
1.43	.9236	.0764	.4236
1.44	.9251	.0749	.4251
1.45	.9265	.0735	.4265
1.64	.9495	.0505	.4495
1.65	.9505	.0495	.4505
1.66	.9515	.0485	.4515
1.95	.9744	.0256	.4744
1.96	.9750	.0250	.4750
1.97	.9756	.0244	.4756



$X = (z*\sigma) + \mu = (1.96 * 100) + 500 = 196 + 500 = 696$

$X = (z*\sigma) + \mu = (1.65 * 100) + 500 = 165 + 500 = 665$

$X = (z*\sigma) + \mu = (1.44 * 100) + 500 = 144 + 500 = 644$

$X = (z*\sigma) + \mu = (1.28 * 100) + 500 = 128 + 500 = 628$

Tier		z-score	Score
First	Top 2.5%	1.96	696
Second	2.5%-5.0%	1.65	665
Third	5.0%-7.5%	1.44	644
Fourth	7.5%-10%	1.28	628